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A thermodynamical model of hadron production in e^+e^- collisions

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Abstract

The hadron production in e^+e^- collisions is studied assuming that particles originate in a hadron gas at thermal and chemical equilibrium. The parameters of the hadron gas are determined with a fit to the average multiplicities of various hadron species measured at LEP and PEP-PETRA colliders. An impressive agreement is found between the predictions of this model and data for almost all particles over a range of production rate of four orders of magnitude. The temperature values found at the centre of mass energies of LEP and PEP-PETRA are around 165 MeV.

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1 Introduction

Hadron production in e^+e^- collisions at high energy is generally believed to be the result of a two-stage process: a parton shower generated by the $q\bar{q}$ pair emerging from the annihilation, and a fragmentation of the partons into observable hadrons. The former process is hard and can be described by perturbative QCD, whereas the latter is soft and not calculable with a perturbative approach.
Several phenomenological models aimed at describing quantitatively the fragmentation process have been developed; amongst them, the most popular are those implemented in the Monte Carlo generators JETSET [1] and HERWIG [2], using the concepts of string and cluster fragmentation respectively. The main unsatisfactory feature of these models, as far as the relative production rates of the hadron species is concerned, is the large number of free parameters required in order to correctly reproduce experimental data. As a consequence, those models have a rather poor predictive power. Empirical regularity in hadron production rates have been observed in ref. [3, 4] although some non-physical assumptions were made about meson quantum numbers and masses. The most interesting result of those observations is that the very same exponential behaviour of the hadron production function is present both at LEP ($\sqrt{s} = 91.2$ GeV) and PEP-PETRA ($\sqrt{s} = 29 \div 35$ GeV) centre of mass energies.
We introduce a thermodynamical approach to the problem of hadronization [5], postulating the existence of a hadron gas in thermodynamical equilibrium before the hadrons themselves decouple (freezing-out) and decay giving rise to observable particles in the detector. We show that this model is able to fit almost all inclusive rates measured so far at LEP and PEP-PETRA colliders in a natural way by using only three parameters. Two of them are the basic parameters of the hadron gas description, namely its temperature T and its volume V; the third one, γ_s is a parameter describing the partial strangeness chemical equilibrium, which has been used already in some analysis of hadron production in heavy-ion collisions [6]. The only required inputs to determine the particle yields are the mass, the spin and the quantum numbers such as baryon number, strangeness, charm and beauty.

2 Hadron gas scheme

Most hadronic events in high energy e^+e^- annihilations are two-jet events. Here we assume that each jet represents a hadron gas phase in complete thermodynamical equilibrium just before the freezing-out time and that the number of such phases in hadronic events is always two, that means neglecting multi-jets events. One can then describe a jet as an object defined by thermodynamical and mechanical quantities such as the temperature and the volume in its rest frame and the Lorentz-boost factor γ. As far as chemical equilibrium is concerned, we assume that, in general, a jet has quantum numbers related to those of the parent quark. As the two jets must be colourless and do not have fractional baryon numbers, sharings of one or several quark-antiquark pairs should occur. It seems reasonable to assume that most pairs are of either $u\bar{u}$ or $d\bar{d}$ type which are the lightest ones. We allow also $s\bar{s}$ sharing in the inter-jet interaction and non-vanishing baryon number for each jet provided that the baryon number and strangeness of the whole system are zero. Morevoer, we assume that each jet keeps the charm and beauty of the parent quark, i.e. no heavy quark pairs are exchanged. Adequate tools to deal with such a problem in a statistical mechanics framework are the canonical partition functions of systems with internal symmetries. The partition function of a system which transforms under the irreducible representation D^ν of a symmetry group G can be expressed as [7, 8]:

$$Z = \frac{d_\nu}{M(G)} \int d\mu(g)\, \chi_\nu(g^{-1})\, \mathcal{Z}(g) \;, \tag{1}$$

where $\mu(g)$ is the group measure, $M(G) = \int d\mu(g)$, d_ν is the dimension of D^ν and $\chi_\nu(g)$ is the character of D^ν, namely $\chi_\nu(g) = \mathrm{tr}\,(D^\nu(g))$. The function $\mathcal{Z}$ is defined as:

$$\mathcal{Z}(g) = \mathrm{tr}\,(e^{-\beta E} e^{-i\sum \phi_i Q_i}) \;, \tag{2}$$

where the ϕ_i's are the parameters of the group and Q_i its generators. The exponential factor is the usual canonical distribution with β as the inverse temperature and E as the energy of the system, both calculated in its rest frame.
In the present case the symmetry group is $U(1)^4$, each $U(1)$ corresponding to the conservation of baryon number N, strangeness S, charm C and beauty B respectively. The Eq. (1) becomes:

$$Z(\mathbf{Q}) = \frac{1}{(2\pi)^4} \int_0^{2\pi}\!\!\int_0^{2\pi}\!\!\int_0^{2\pi}\!\!\int_0^{2\pi} d^4\phi\; e^{i\mathbf{Q}\cdot\phi}\, \mathcal{Z}(\phi) \;. \tag{3}$$

$\mathbf{Q} = (N, S, C, B)$ is a four dimensional vector with integer components representing the quantum numbers of a jet and $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$, each ϕ_i being the parameter of a $U(1)$ group.
According to the assumptions made for the quantum numbers of the system, the partition function of the whole system can be written as:

$$\hat{Z} = \sum_{N=0}^{1} \sum_{S=-2}^{2} Z(\mathbf{Q}) Z(-\mathbf{Q}) \;, \tag{4}$$

where $Z(\mathbf{Q})$ are the partition functions of the single jets and the sum runs over baryon number N and strangeness S. Further constraints are implicitly introduced in Eq. (4), namely a maximum allowed baryon number 1 and a maximum allowed strangeness 2. Moreover, in case of $e^+e^- \to s\bar{s}$ events the term with $S = 0$ is removed from the sum. These assumptions define what has been called correlated jet scheme [5]. For a gas of N_B boson species and N_F fermion species, the partition function of a single jet becomes:

$$\begin{aligned} Z(\mathbf{Q}) \;\; &= \;\; \frac{1}{(2\pi)^4} \int d^4\phi\; e^{i\mathbf{Q}\cdot\phi} \exp\{\sum_{j=1}^{N_B} \sum_k \log\,(1 - e^{-\beta\varepsilon_k - i\mathbf{q}_j\cdot\phi})^{-1} + \\ &+ \sum_{j=1}^{N_F} \sum_k \log\,(1 + e^{-\beta\varepsilon_k - i\mathbf{q}_j\cdot\phi})\} \;, \end{aligned} \tag{5}$$

where k labels all available states of phase space for the j^{th} particle, ε_k is its energy and $\mathbf{q}_j = (N_j, S_j, C_j, B_j)$. Since the total number of each hadron in the gas is not determined, all chemical potentials must vanish. From Eq. (4) we are able to estimate the average number per jet of any hadronic particle assigning each of them a fictitious fugacity λ_i which multiplies the $e^{-\beta\varepsilon_k}$ factors and deriving:

$$< n_i >= \lambda_i \frac{\partial \log \hat{Z}}{\partial \lambda_i}|_{\lambda_i = 1} \;. \tag{6}$$

The sum over the phase space, for a continuous level density becomes:

$$\sum_k \longrightarrow (2J+1)\frac{V}{(2\pi)^3}\int d^3p \; , \tag{7}$$

where J is the spin of the particle and V the volume of the hadron gas phase.
One is left with complicated integrals in Eq. (5),(6) but, if $T \sim \mathcal{O}(100)$ MeV, as we can argue from the QCD soft scale, the exponential factors are expected to be very small for all particles but pions, so that

$$\begin{aligned}
&\log\left(1 \pm e^{-\sqrt{p^2+m_i^2}/T - i\mathbf{q}_i\cdot\phi}\right)^{\pm 1} \simeq e^{-\sqrt{p^2+m_i^2}/T - i\mathbf{q}_i\cdot\phi} \\
&\frac{1}{e^{\sqrt{p^2+m_i^2}/T + i\mathbf{q}_i\cdot\phi} \pm 1} \simeq e^{-\sqrt{p^2+m_i^2}/T - i\mathbf{q}_i\cdot\phi}
\end{aligned} \tag{8}$$

are very good approximations. This corresponds to the Boltzmann limit of Fermi and Bose statistics. It can be shown then that the average number of particles i produced per jet is

$$< n_i >= z_i \frac{Z(\mathbf{Q}-\mathbf{q}_i)}{Z(\mathbf{Q})} \; , \tag{9}$$

whereas for pions

$$< n_i >= \frac{V}{(2\pi)^3}\int d^3p \; \frac{1}{e^{\sqrt{p^2+m_i^2}/T} - 1} \; , \tag{10}$$

being

$$z_i = (2J_i+1)\frac{V}{(2\pi)^3}\int d^3p \; e^{-\sqrt{p^2+m_i^2}/T} = (2J_i+1)\frac{VT}{2\pi^2} m_i^2 K_2(\frac{m_i}{T}) \; . \tag{11}$$

K_2 is the modified Bessel's function of order 2.
The factor $Z(\mathbf{Q}-\mathbf{q}_i)/Z(\mathbf{Q})$ suppresses or enhances the thermal production rate of the particle according to its quantum numbers and the quantum numbers of the jet (see also Fig. 1).

3 Fit of LEP and PEP-PETRA data

The volume and the temperature of the two phases at the decoupling time are determined by a fit to the available data on hadron inclusive production. The volume and the temperature are assumed to be equal for both jets and also equal for different kinds of primary quarks. A parameter $\gamma_s < 1$ is introduced in order to take into account a non-complete strange chemical equilibrium; if a hadron contains n strange valence quarks, its production rate is reduced by a factor γ_s^n. This reduction applies also to neutral mesons such as η, η', ϕ, ω according to the fraction of $s\bar{s}$ content in the meson itself.
The hadron rates calculation proceeds through two steps; in the first one a primary number of hadrons emitted from the thermal source is calculated according to the formulae of previous section; in the second, all decays of these primary hadrons are performed according to known decay modes and branching ratios. The overall production of each particle is then computed by adding up the fraction stemming from decays of heavier particles to the primary one. The decay chain stops when π, K, K_L^0, μ or stable particles are reached, in order to match the numbers of production rates provided by experiments at e^+e^- colliders, including all decay products of

particles with $c\tau < 10$ cm. All hadrons included in JETSET 7.4 tables have been considered, with the quoted masses and widths. All other light flavoured resonances up to a mass of 1.7 GeV have been included with masses, widths and branching ratios quoted by PDG [9]. The mass of resonances with $\Gamma > 1$ MeV has been distributed according to a relativistic Breit-Wigner function within $\pm 2\Gamma$ from the central value. Experimental production rates used in the fit at $\sqrt{s} = 91.2$ GeV [10] and at $\sqrt{s} = 29 \div 35$ GeV [9] are weighted averages of all available measurements.

The results of the fit are summarized in Tables 1, 2 and Figs. 2, 3. The errors on the parameters T, V and γ_s estimated from the fit include also uncertainties on input data such as branching ratios, masses and widths of particles involved in the fit. The fit procedure is described more in detail in ref. [5].

An excellent agreement between experimental and calculated production rate values is achieved. A significative deviation is observed at $\sqrt{s} = 91.2$ GeV for the Σ^* production rate whose experimental value is still controversial because of the large disagreement between different experiments [11].

By using as input data the fitted T, V and γ_s values, the model also yields a definite prediction for the relative production rates of heavy flavoured hadrons once $R_c = \sigma(e^+e^- \to c\bar{c})/\sigma(e^+e^- \to hadrons)$ and $R_b = \sigma(e^+e^- \to b\bar{b})/\sigma(e^+e^- \to hadrons)$ are known.

In Table 3 the predicted multiplicities of some of the lightest heavy flavoured hadrons are quoted together with corresponding experimental vaues [12]. A very good agreement with data is found. Two main sources of systematic uncertainties have been considered. The first one is simply a possible systematic error in the parameters fit which has been estimated by excluding the data points deviating the most from theoretical values and repeating the fit itself. The second one is related to possible different values of V as a function of the primary quark mass. This has been taken into account by introducing two dimensionless variables $x_c, x_b \in [0,1]$ such that $x_c V$ is the volume in $e^+e^- \to c\bar{c}$ and $x_b V$ in $Z \to b\bar{b}$ events, whereas $V(1 - x_c R_c - x_b R_b)/(R_u + R_d + R_s)$ is the volume for $Z \to q\bar{q}$ where q is a light quark. The measurement of the averaged charged multiplicity in $e^+e^- \to c\bar{c}$ and $e^+e^- \to b\bar{b}$ [13, 14] determine $x_c = 0.88 \pm 0.03$ and $x_b = 0.70 \pm 0.016$ at $\sqrt{s} = 91.2$ GeV and $x_c = 0.89 \pm 0.12$ and $x_b = 0.47 \pm 0.15$ at $\sqrt{s} = 29 \div 35$ GeV. Inserting those values in the fit, it is found that new fitted parameters are almost unchanged. Other systematic effects related to uncertainties on R_q are found to be negligible. The variations of the fitted parameters due to those mentioned effects are summarized in Table 4.

Another important issue is the independence of the fitted parameters, in particular T and γ_s, from the neglected part of the hadronic mass spectrum, i.e. light-flavoured resonances with mass > 1.7 GeV. This has been tested by moving the cut-off point down to 1.3 GeV by steps of 0.1 GeV. The new values are found to be consistent with the main quoted values within the fit error, as shown in Fig. 4. Morevoer, the number of primary hadrons stabilizes around 17 as the cut-off points increases. This indicates that missing heavy resonances production is negligible and should not affect significantly the fit.

One of the most relevant results is that temperatures values at both centre of mass energies turn out to be very close. If we add in quadrature the systematic uncertainties quoted in Table 4 to the fits error quoted in Table 1 we get:

$$T_{LEP} = 162.8 \pm 2.9 \quad (12)$$

$$T_{PEP} = 169.5 \pm 3.7 \,. \quad (13)$$

The χ^2 of the weighted average of those values is 1.8, which indicates a good degree of compatibility. Also, it should be taken into account that the temperature value of PEP-PETRA might be affected by an additional systematic error due to having averaged production rates measurements spread between 29 and 35 GeV.

4 Conclusions

The problem of the relative production of hadrons in e^+e^- collisions is treated with a thermodynamical approach postulating that hadronic jets must be identified with hadron gas phases in thermodynamical equilibrium before the hadrons themselves decouple and subsequently decay into lighter particles. Nevertheless, since the particle production depends almost linearly on the volume, the occurrence of additional jets, i.e. phases, in the same event should affect the relative hadron production rates very mildly provided that the temperature of the additional jets is the same.

It has been shown that this model is able to fit very well the average multiplicities of light hadrons per hadronic event observed at LEP ($\sqrt{s}$=91.2 GeV) and PEP-PETRA ($\sqrt{s} = 29 \div 35$ GeV) for a variation of production rate by four orders of magnitude, from π to Ω.

The predicted rates of heavy flavoured hadrons are also in good agreement with data. Only three parameters are required in order to reproduce data correctly, namely the temperature and the volume of the hadron gas and a parameter $\gamma_s \in [0, 1]$ allowing an incomplete strange chemical equilibrium. The temperature values at $\sqrt{s} = 91.2$ GeV and $\sqrt{s} = 29 \div 35$ GeV, determined with the fit, are very similar and indicate a constant hadronization temperature. The thermalization of the system could be a characteristic of the quark-hadron transition, brought about by strong interactions.

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Figure captions

Figure 1 Behaviour of the jet partition function Z as a function of baryon number N for $(S, C, B) = 0$ (left) and as a function of the strangeness S for $(N, C, B) = 0$ (right), for $T = 163$ MeV and $V = 21$ Fm^3. The suppression factors $Z(\mathbf{Q} - \mathbf{q}_i)/Z(\mathbf{Q})$ are smaller for increasing N and S.

Figure 2 Fit of hadron production measured at LEP. Above: the solid line connects fitted values while data are shown as black dots. Below: fluctuations of measured points from the fitted values (solid line) in standard deviation units. Error bars include also contributions from uncertainty on masses, widths and branching ratios of hadrons.

Figure 3 Fit of hadron production measured at PEP-PETRA. Above: the solid line connects fitted values while data are shown as black dots. Below: fluctuations of measured points from the fitted values (solid line) in standard deviation units. Error bars include also contributions from uncertainty on masses, widths and branching ratios of hadrons.

Figure 4 Fitted temperature T, volume V, γ_s and number of primary hadrons at $\sqrt{s} = 91.2$ GeV as a function of the light flavored resonance cut-off mass.

Tables

Parameters	$\sqrt{s} = 91.2$ GeV	$\sqrt{s} = 29 \div 35$ GeV
Temperature(MeV)	162.9±2.1	169.3±3.5
Volume(Fm^3)	21.4±1.9	9.3±1.4
γ_s	0.696±0.027	0.811±0.046
χ^2/dof	38.6/17	29.3/12

Table 1: Fit results.

	$\sqrt{s} = 91.2$ GeV		$\sqrt{s} = 29 \div 35$ GeV	
Hadrons	Calculated	Measured	Calculated	Measured
π^0	10.03	$9.19 \pm 0.73 \pm 0.37$	6.186	$5.6 \pm 0.3 \pm 0.22$
π^+	8.952	$8.53 \pm 0.22 \pm 0.34$	5.415	$5.15 \pm 0.2 \pm 0.21$
K^+	1.065	$1.18 \pm 0.064 \pm 0.013$	0.750	$0.74 \pm 0.045 \pm 0.008$
K^0	1.010	$1.006 \pm 0.016 \pm 0.013$	0.704	$0.74 \pm 0.035 \pm 0.008$
η	0.866	$0.94 \pm 0.11 \pm 0.11$	0.535	$0.61 \pm 0.07 \pm 0.0056$
ω	0.939	$1.11 \pm 0.14 \pm 0.011$	0.544	–
ρ^0	1.186	$1.29 \pm 0.13 \pm 0.17$	0.704	$0.81 \pm 0.08 \pm 0.11$
K^{*+}	0.350	$0.357 \pm 0.034 \pm 0.089$	0.246	$0.32 \pm 0.025 \pm 0.006$
K^{*0}	0.344	$0.379 \pm 0.020 \pm 0.089$	0.241	$0.28 \pm 0.03 \pm 0.006$
p	0.534	$0.489 \pm 0.05 \pm 0.041$	0.293	$0.32 \pm 0.025 \pm 0.015$
η'	0.0993	$0.22 \pm 0.07 \pm 0.0002$	0.0702	$0.26 \pm 0.1 \pm 0.00007$
ϕ	0.123	$0.107 \pm 0.009 \pm 0.002$	0.106	$0.085 \pm 0.011 \pm 0.002$
Λ	0.162	$0.185 \pm 0.007 \pm 0.007$	0.104	$0.103 \pm 0.005 \pm 0.005$
$(\Sigma^+ + \Sigma^-)/2$	0.0366	$0.044 \pm 0.007 \pm 0.0035$	0.0226	–
Σ^0	0.0383	$0.036 \pm 0.007 \pm 0.0035$	0.0243	–
Δ^{++}	0.0925	$0.062 \pm 0.032 \pm 0.011$	0.0488	–
Ξ^-	0.0122	$0.0128 \pm 0.0007 \pm 0.0004$	0.00893	$0.0088 \pm 0.0014 \pm 0.0003$
$(\Sigma^{+*} + \Sigma^{*-})/2$	0.0192	$0.011 \pm 0.002 \pm 0.0014$	0.0111	$0.0085 \pm 0.002 \pm 0.0015$
Ξ^{*0}	0.00421	$0.0031 \pm 0.0006 \pm 0.00007$	0.00279	–
Ω	0.000856	$0.00080 \pm 0.00025 \pm 0.0001$	0.000659	$0.007 \pm 0.0035 \pm 0.00015$

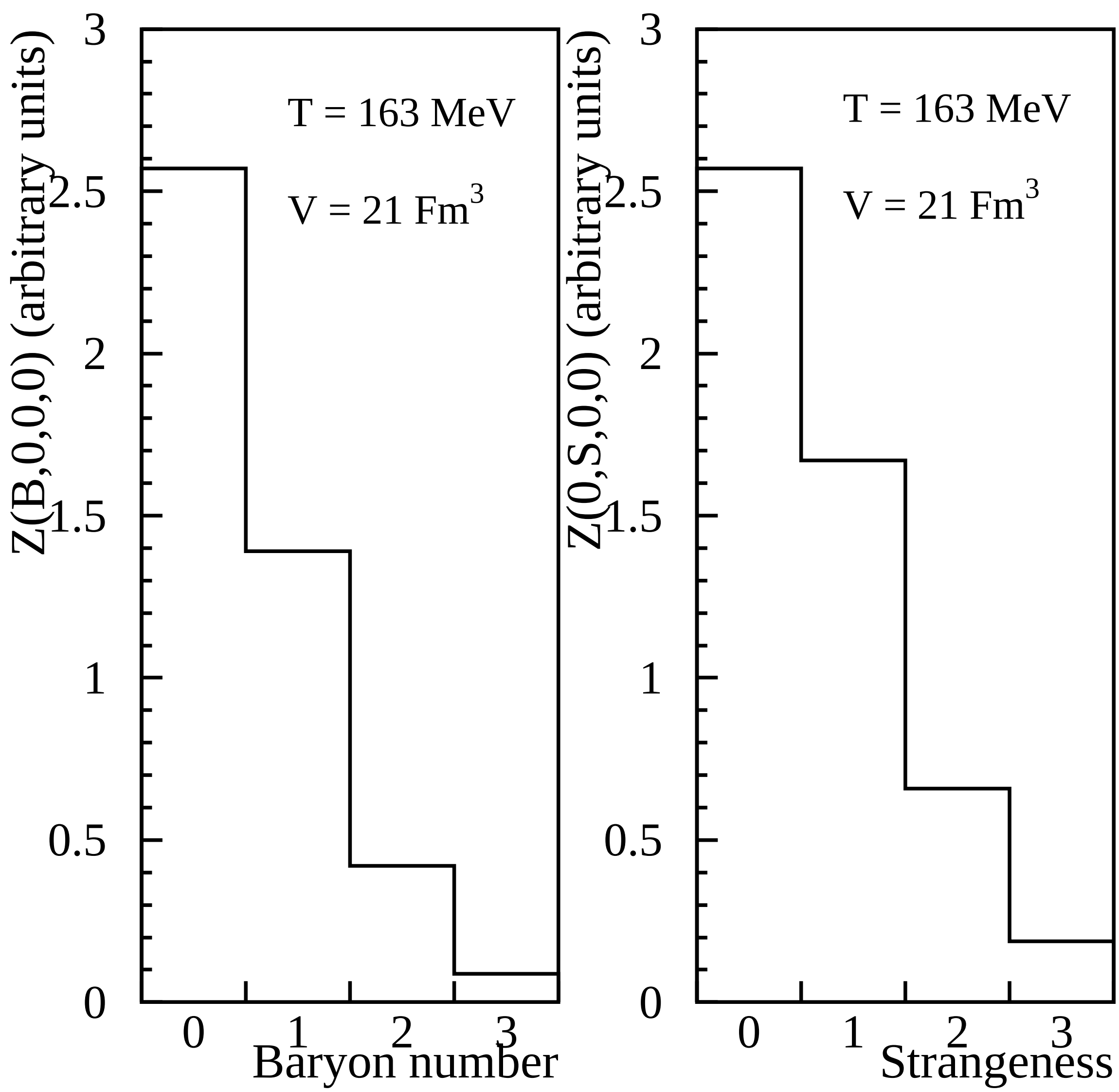
Table 2: Measured hadrons average multiplicities per hadronic event compared to the fitted values. The first error is the experimental one, the second error is due to uncertainty on hadron masses, widths and branching ratios.

	$\sqrt{s} = 91.2$ GeV		$\sqrt{s} = 29 \div 35$ GeV	
Hadrons	Calculated	Measured	Calculated	Measured
D^0	0.231	0.221 ± 0.012	0.256	0.225 ± 0.035
D^+	0.0914	0.087 ± 0.008	0.106	0.085 ± 0.015
D^{*+}	0.107	0.088 ± 0.0056	0.113	0.192 ± 0.036
B	0.0902	0.097 ± 0.026	0.0387	–
B^{*0}/B^0	0.694	0.745 ± 0.069	0.697	–
Λ_c^+	0.0311	0.037 ± 0.009	0.0339	0.055 ± 0.025
D_s	0.0536	0.041 ± 0.0076	0.0542	–

Table 3: Measured heavy flavoured hadrons average multiplicities per hadronic event compared to predictions of the model. The experimental values [12] have been averaged according to the procedure of ref. [15]. Values of $R_c = 0.17$, $R_b = 0.22$ at $\sqrt{s} = 91.2$ and of $R_c = 0.36$, $R_b = 0.09$ at $\sqrt{s} = 29 \div 35$ have been used.

Parameter	$\sqrt{s} = 91.2$ GeV	$\sqrt{s} = 29 \div 35$ GeV
Temperature(MeV)	2.06	1.08
Volume(Fm^3)	1.34	0.47
γ_s	0.017	0.018

Table 4: Systematic uncertainties on fitted parameters.



Z(B,0,0,0) (arbitrary units)
T = 163 MeV
V = 21 Fm^3
3
2.5
2
1.5
1
0.5
0
0
1
2
3
Baryon number
Z(0,S,0,0) (arbitrary units)
T = 163 MeV
V = 21 Fm^3
3
2.5
2
1.5
1
0.5
0
0
1
2
3
Strangeness

Figure 1:

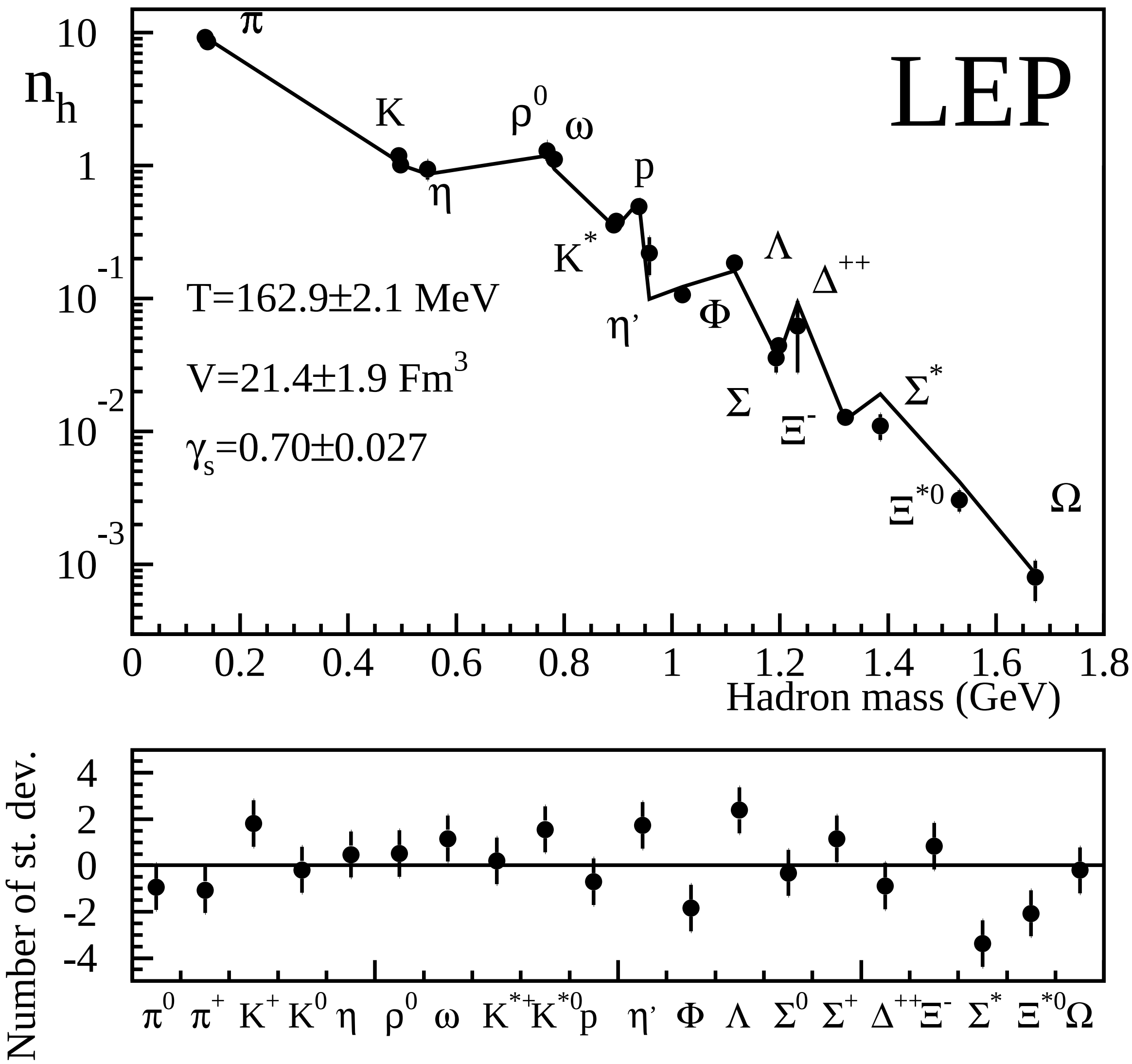
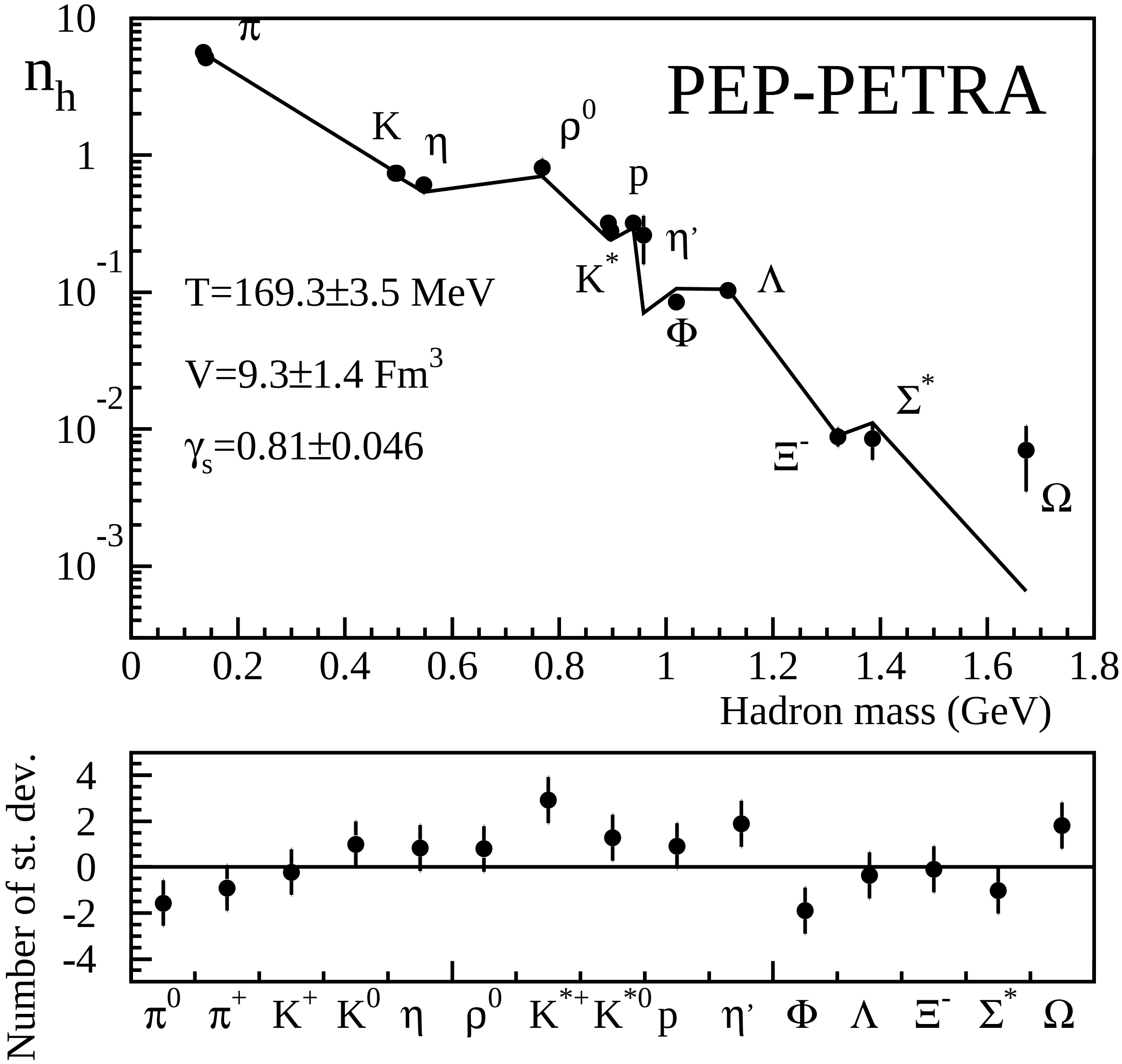
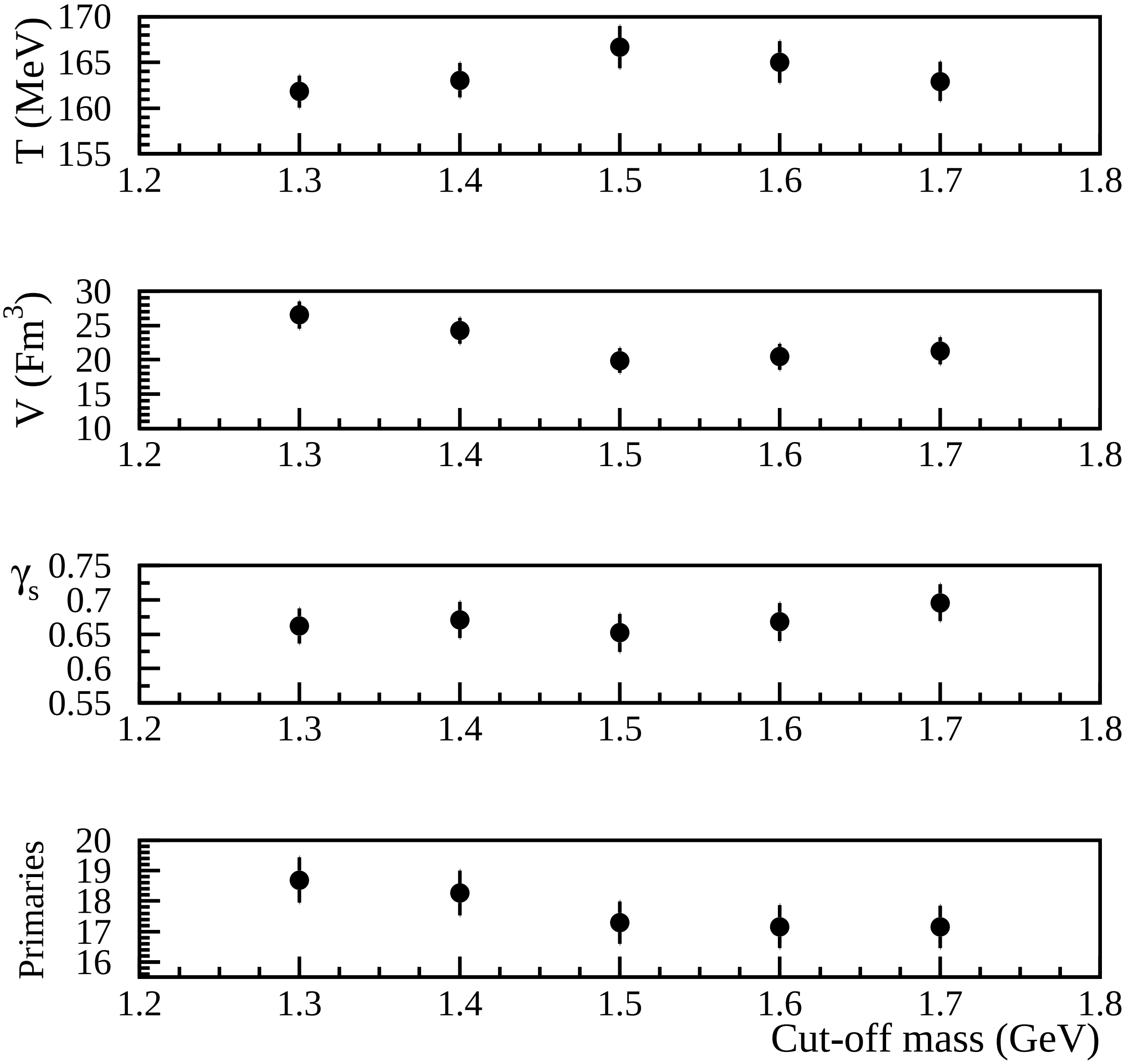

LEP
n_h
T=162.9±2.1 MeV
V=21.4±1.9 Fm^3
γ_s=0.70±0.027
π
K
η
ρ^0
ω
p
K^*
η'
Φ
Λ
Δ^++
Σ
Ξ^-
Σ^*
Ξ^*0
Ω
10
1
10^-1
10^-2
10^-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Hadron mass (GeV)
Number of st. dev.
4
2
0
-2
-4
π^0 π^+ K^+ K^0 η ρ^0 ω K^*+ K^*0 p η' Φ Λ Σ^0 Σ^+ Δ^++ Ξ^- Σ^* Ξ^*0 Ω

Figure 2:



PEP-PETRA
n_h
T=169.3±3.5 MeV
V=9.3±1.4 Fm^3
γ_s=0.81±0.046
Hadron mass (GeV)
Number of st. dev.

Figure 3:



T (MeV)
170
165
160
155
1.2
1.3
1.4
1.5
1.6
1.7
1.8
V (Fm^3)
30
25
20
15
10
γ_s
0.75
0.7
0.65
0.6
0.55
Primaries
20
19
18
17
16
Cut-off mass (GeV)

Figure 4: